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Reliability Analysis of Complex Systems: An Improved Risk Importance Measure Approach

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Abstract: *This paper develops a new feature of the risk achievement worth (RAW) measure to expand its application to complex transportation directed networks, rather than being limited to traditional or undirected networks. The idea is to combine the RAW measure with modern techniques using the NetworkX library in Python, to improve its computational capabilities and handle complex directed networks efficiently. The approach was applied to a complex system . We calculated its reliability using a polynomial approach based on minimal paths and minimal cut sets, in addition to calculating the probability of path success and the probability of failure for cut sets. To demonstrate the effectiveness of the risk achievement worth measure, it was compared with the Birnbaum measures for analyzing the importance of network components at a default value of R = 0.60. The results showed complete consistency in terms of the importance levels of both measures, as confirmed by the Spearman correlation coefficient, with a value* $\boldsymbol{S_R = 1}$. *Developing the risk achievement worth for analyzing complex networks is crucial to overcoming the computational complexity of the Birnbaum measure, which relies on differentials, unlike the RAW measure, which relies on simplified arithmetic procedures independent of differentials. Therefore, this provides an enables rapid identification of critical vulnerabilities in large-scale infrastructures.*

Keywords: *Complex system, risk achievement worth measure , NetworkX library, Birnbaum measure*

Introduction

The general framework for the concept of complex network reliability is the probability that the network will perform its assigned tasks, meet challenges, and continue to perform without interruption. Therefore, reliability analysis is critical to mitigating outages and ensuring service continuity. With the advancement of technology, there is an urgent need to study the reliability of complex systems and strive to increase their reliability, especially those related to energy, communications, and transportation systems. The study of complex systems seeks to understand the nature of systems and how their components interact to achieve desired outcomes. To achieve this, appropriate tools must be developed to approach systems comprehensively from a mathematical perspective, as is the case in network design.

Graph theory is the foundation for establishing design and innovation principles for systems, regardless of their complexity through the interconnections between system components, a connectivity matrix can be defined, providing a starting point for discovering component connections and locations [1]. Previous studies have seen significant progress in the study of reliability analysis of complex systems or methods for their design. The first of these studies was by Moore and Shannon, who focused their work on probability theory in network modeling to understand the communication behavior between network nodes [2-5]. In their quest to understand the mechanism of flows resulting from interactions within network components, the maximum flow algorithm, invented in 1962, emerged as an excellent tool for identifying minimal discontinuities, [6-9]. However, this algorithm did not adequately address the need for reliable system optimization Because of its inability to prioritize the importance levels of system components. Therefore, an effective theoretical foundation was needed to uncover the nature of the network and its components, and which parts required periodic maintenance to mitigate system failures or vulnerabilities.

The Birnbaum measure emerged in 1969, offering a glimmer of hope for the further development of reliability analysis. This measure defined a mathematical framework for delving into the details of the network, determining the importance of each component, and determining which has the greatest impact on network reliability, [10-13]. Over the years, networks have become more complex, increasing the requirements for calculating network reliability. The inclusion-exclusion methodology, devised in 1995, effectively addressed large, highly complex networks, [14,15]. With the growing complexity of network design and the multiplicity of its tasks, it became necessary to establish integrated foundations for analyzing network reliability based on the criticality of its components to ensure their performance. Therefore, the Birnbaum measure faced challenges resulting from this complexity, as its calculations are based on the partial derivative between system reliability and component reliability.

The measure of risk achievement worth (RAW) was designed to assume this task by determining the criticality of network components and measuring the potential impact on network reliability in the event of the failure of one of its components [16]. Compared to the Birnbaum measure, the RAW measure has less complex calculations because it is not based on differential calculations, which have achieved significant effectiveness in its applications. However, the effectiveness of the RAW measure has been limited due to its applications within simple or undirected traditional networks, such as infrastructure networks or nuclear reactors, due to the limited logic of this measure [17]. Hence, this study aims to develop this measure to include complex directed networks in its applications. This measure has been integrated with modern software using the NetworkX library in Python and applied to a complex network system, where it achieved promising results when compared to the Birnbaum measure. This work aims to establish the foundations for modeling complex networks, paving the way for guiding the optimization processes of complex systems with complete skill and knowledge.

Complex Network System

The complex transportation network system as shown in Figure 1. is the system of interest in this study. This system is represented by a directed graph $G(V,E)$, where nodes $v_i, i = 1, \dots, 11$ represent links between components, and edges represent the c_i components, and v_1 is the source and v_{11} the output nodes. Each component $c_i \in E$ in the network has a probability of success R_c and a probability of failure $1 - R_c$. The goal is to calculate system reliability R_t and accurately determine component importance levels, ensuring continuous system performance.

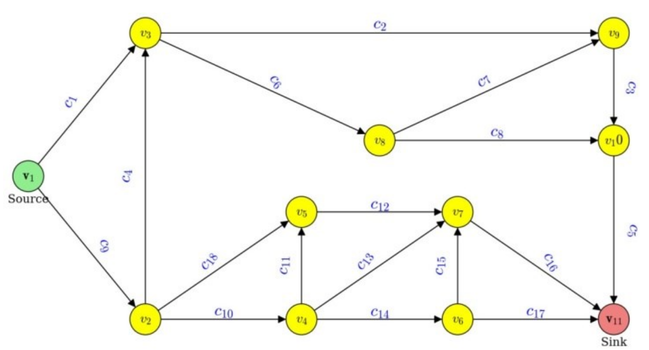

Figure 1. Complex system network modeling.

Mathematical methodology

An integrated mathematical approach was followed to determine the reliability value of the system, relying on two methods, one of which is

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based on finding the minimal paths, and the other is based on the minimal cut, according to the following two theorems:

Theorem 1. [18,19]. let $MP_1, MP_2, \dots, MP_n$ denote all minimal paths of complex network G. Then, the reliability system R_t of G is given by:

$$R_t = \prod_{i=1}^{n} (1 - \prod_{R_j \in MP_i} R_j)$$

The proposed method is to find the reliability of the system by finding minimal cut and then applying Theorem 2:

Theorem 2. let $MC_1, MC_2, \dots, MC_n$ denote all minimal cut sets of complex network G. Then, the reliability system R_t of G is given by:

$$R_t = \prod_{i=1}^{n} (1 - \prod_{R_j \in Mc_i} (1 - R_j))$$

The novelty of the methodology used in this study lies in the implementation of the NetworkX library in Python, starting with its use in network modeling and finding minimal paths using the (all simple paths) function, in addition to its important role in calculating reliability based on Theorems 1 and 2. Furthermore, the Birnbaum and risk achievement worth (RAW) measures are implemented using this modern library, particularly with the (RAW) measure, as most studies indicate that this metric is implemented using expensive classical analytical methods. The mathematical formula for these measures is as follows:

$$\text{Importance}_B\ (c_i) = \frac{\partial R_t}{\partial R_i}, i = 1, \dots, n$$

$$Importance_{RAW}\ (c_i) = R(t) - R(t \backslash c_i)$$

where $t \backslash c_i$ is the system given component c_i fails. In addition, the correlation coefficient of these two measures is calculated using the Spearman formula:

$$S_R = 1 - \frac{6 \sum_{i=1}^{n} d_i^2}{n.(n^2 - 1)}$$

where d_i is the difference value between the c_i component of both measures.

Implementation and Discussion

This section covers the application of the methodology and includes two main parts: the first concerns the calculation of system reliability using a polynomial approach, and the second concerns the application of importance metrics to the network.

A. Calculation of System Reliability

The following are the details of the minimal network paths shown in Figure 1, where each number represents a component index:

$$MPS = \{(1,2,3,5), (1,6,7,3,5), (1,6,8,5), (9,4,2,3,5), (9,4,6,7,3,5), (9,4,6,8,5), (9,10,11,12,16), (9,10,13,16), (9,10,14,15,16), (9,10,14,17), (9,18,12,16)\}$$

These minimal paths were found by implementing the NetworkX library in Python using the (all simple paths) function. To perform a more advanced analysis of the network's reliability, the success probability of each path was extracted based on the assumption that each component has an independent success probability, with default values ranging from 0 to 1. An R-value of 0.5 was chosen for all components. This approach allows the reliability of each path to be evaluated under equal conditions, as in significance or sensitivity analysis. This approach can play an important role in guiding improvement actions to address weaknesses in components with low reliability values. Path success was calculated according to the following mathematical formula:

$$\Pr(\mathrm{MP_i}) = \prod_{c\in \mathrm{MP_i}} \mathrm{R_c},$$

where $\mathrm{MP_i} = \{\mathrm{MP_1}, \mathrm{MP_2}, \ldots\ldots \mathrm{MP_n}\}$. The following Table.1 summarizes the details of the minimal paths in terms of path size, number of paths, and the range of success probabilities.

Table 1. Distribution of minimal paths by size with samples of them and the most frequent components, in addition to the success range probabilities.

Minimal Path size	Number of Minimal Paths	Range of Success R=0.5
4	5	0.0625
5	5	0.0312
6	1	0.0156

We chose the value R=0.5, since it is the worst possible scenario for operating the system.

Figure 2 shows the distribution of paths with the probability of success (P) for each group according to the data in Table 1.

Figure 2. Visualize the minimal Path distribution by size with a range of success probabilities.

To find the minimal cut sets, the following steps will be followed, for more detail see [20]:

Step 1. Create a connection matrix.

Step 2. Finding Source and Destination Cuts

Step 3. Create and filter column Combinations

Step 4. Create and filter column combinations for all remaining combinations.

By applying the above steps, the network shown in Figure 1 will contain at least 60 minimal cut sets. The probability of failure of each minimum component was calculated based on the assumption that each component has an independent probability of failure, with default values ranging from 0 to 1. An R-value of 0.5 was chosen for all components. This approach allows for the reliability of each component to be evaluated under equal conditions, helping to identify the components most in need of continuous improvement, which in turn contributes to network optimization. The probability of a component ($\mathrm{mc_i}$) failure was calculated according to the following mathematical formula:

$$\Pr(\mathrm{MC_i}) = \prod_{c\in \mathrm{MC_i}} (1 - \mathrm{R_c}),$$

where $\mathrm{MC_i} = \{\mathrm{MC_1}, \mathrm{MC_2}, \ldots\ldots \mathrm{MC_n}\}$.

The following Table 2 summarizes the details of the minimal cut sets in terms of cut size, number of cuts, examples, and the failure probability range when R = 0.5 :

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Table 2. Distribution of minimal cut sets by size with failure probability range

Cut Set Size	Number of Minimal Cut Sets	Failure Probability Range
2	2	0.25
3	8	0.125
4	21	0.0625
5	12	0.0321
6	10	0.0156
7	6	0.0078
8	1	0.0039

Figure 3 shows the distribution of minimal cut sets by size with failure probability range according to the data in Table 2.

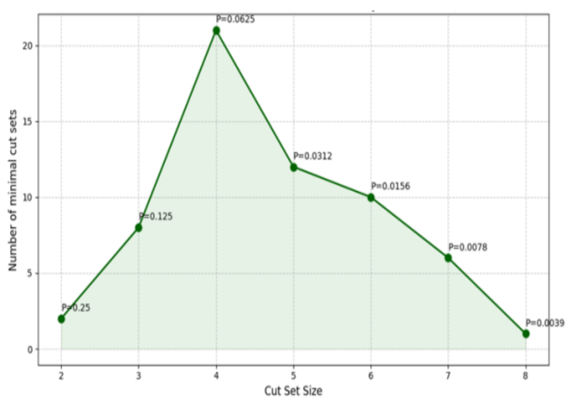

Figure 3. Visualize the minimal cut sets by size with failure probability range

When the minimal paths are substituted into the reliability function equation given in Theorem 1 and the minimal cut sets into the reliability function equation given in Theorem 2 and replace the symbol c with the symbol R, the system reliability function R_t produces the same 669-term polynomial:

Note: Consider that if a component in the network succeeds, its reliability is equal to one, and if it fails, it is equal to zero. To simplify polynomials, we will use Boolean algebra, where $R_i^n = R_i \; \forall n$, [21, 22].

$$R_t = R_1R_{10}R_{11}R_{12}R_{13}R_{14}R_{15}R_{16}R_{17}R_{18}R_2R_3R_4R_5R_6R_7R_8R_9 - \cdots + R_4R_5R_6R_8R_3$$

B. Importance Measures in System Reliability

In this section, the Birnbaum and risk achievement worth (RAW) measures will be applied to calculate the importance of network components and compare the results. Then, the correlation coefficient between these two scales was calculated. The measures were tested on a single default value of R = 0.60 to ensure their significance levels were evaluated.

Table 3. Distribution of minimal cut sets by size with failure probability range

C_I	Birnbaum Measure	(RAW) Measure
c_9	0.4307	0.2584
c_5	0.3083	0.1850
c_1, c_{10}	0.1927	0.1156
c_{16}	0.1952	0.1171
c_3, c_6	0.1307	0.0784
$c_2 c_8$	0.0881	0.0529
c_{14}, c_{12}	0.0838	0.0503
c_{17}, c_{18}	0.0662	0.0397
c_4	0.0452	0.0271
c_{13}	0.0364	0.0218
c_7	0.017	0.0102
c_{15}, c_{11}	0.007	0.0042

As shown in Table 3, the component importance levels are consistent across the same default value R = 0.60, enhancing the metrics' ability to accurately determine component importance levels. Also, the results confirm that the NetworkX library in Python has successfully implemented this metric accurately on directed

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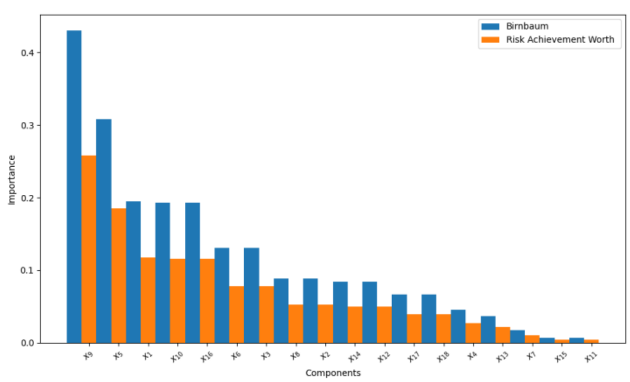
networks, a first for any such implementation, making it a viable alternative to the Birnbaum measure. Furthermore, the risk achievement worth measure is easy to compute, unlike the computational complexity of the Birnbaum measure, which is further complicated by differentiation processes in large networks. These measures are a powerful tool for ensuring system reliability and accurately guiding network optimization without dispersing resources.

The correlation coefficient values for the results in Table 3 using Spearman's rank correlation coefficient formula confirm complete consistency between the measures, as the correlation coefficient value matches the value 1. This is due to the similarity of the order of levels for both measures, which results in ($\sum_{i=1}^{n18} d_i^2 = 0$) as follows:

$$S_R = 1 - \frac{6\sum_{i=1}^{n18} d_i^2}{18.(18^2 - 1)}$$



$$S_R = 1 - \frac{6.(0)}{18.(18^2 - 1)} = 1$$

Figure 4. Visualize the minimal cut sets by size with failure probability range

The visual representation in Figure 4 clearly shows the consistency of the levels of both measures for all hypothetical values of $R = 0.60$. Despite the differences in values, the consistency of the levels remained constant, with the importance values of the RAW measure being lower than those of the Birnbaum measure, confirming the effectiveness of the approach taken to develop the RAW measure to be a strong competitor to the Birnbaum measure.

Conclusions

The integration of the RAW measure with the NetworkX library has enhanced the efficiency of the RAW measure in dealing with complex routed networks. The Spearman correlation coefficient of 1 for both the RAW and the Birnbaum measures confirms that the RAW measure is a strong competitor to the Birnbaum metric, as its calculations do not rely on differential computational complexity. Furthermore, it validates the approach adopted. The NetworkX library facilitates the acceleration of finding minimal paths using the (all simple paths) function, which also speeds up polynomial calculations, providing accurate system reliability results without approximation. The RAW measure contributes to improving the reliability of complex routed networks by directing the optimization process to relatively less reliable components, ensuring continued system efficiency. Finally, the availability of tools that enable accurate network analysis enables system engineers and designers to reliably expand their vision of network applications.

Conflicts of interest

The author declares no conflict of interest.

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