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Extended Kalman Filter with Optimal Control On Dengue Model

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Abstract: In real life, due to various measurement limitations, not all variables in the dengue fever epidemic model can be measured. Therefore, a tool is needed to estimate unmeasured variables with known related variables. One method for estimating variables in nonlinear systems is the extended Kalman filter (EKF). Next, using these estimated results, optimal control will be sought in the form of vaccination to reduce the number of infections. From the simulation results, it can be concluded that state estimation with EKF for the dengue fever model is good enough to estimate state that are disturbed by a random variable within the selected range of disturbance covariance. Then, it was found that the smaller the standard deviation of the disturbance, the smaller the optimal control required at the start. Thus, the greater the disruption, the greater the costs spent.

Keywords: *EKF, optimal control, dengue, vaccination*

Abstrak: Dalam kehidupan nyata, karena berbagai keterbatasan pengukuran, tidak semua variabel dalam model epidemi demam berdarah dapat diukur. Oleh karena itu diperlukan suatu alat untuk mengestimasi variabel-variabel yang tidak terukur dengan variabel-variabel terkait yang diketahui. Salah satu metode untuk memperkirakan variabel dalam sistem nonlinier adalah *Extended Kalman Filter* (EKF). Selanjutnya, dengan menggunakan hasil perkiraan tersebut, akan dicari pengendalian yang optimal berupa vaksinasi untuk menurunkan jumlah infeksi. Dari hasil simulasi dapat disimpulkan bahwa estimasi keadaan dengan EKF untuk model demam berdarah cukup baik untuk mengestimasi setiap variabel dengan pengukuran yang diganggu oleh variabel acak dalam rentang kovarians gangguan yang dipilih. Kemudian ditemukan bahwa semakin kecil standar deviasi gangguan maka semakin kecil pula pengendalian optimal yang diperlukan pada saat start. Dengan demikian, semakin besar gangguan maka semakin besar pula biaya yang dikeluarkan.

Kata Kunci: *E*KF, kontrol optimal, demam berdarah, vaksinasi

Introduction

Dengue is an infectious disease caused by the dengue virus which is transmitted through the bite of the *Aedes aegypti* mosquito. This disease is a significant public health problem in many tropical and subtropical countries, including Indonesia[1]. Dengue control efforts include various approaches, from vector (mosquito) control to vaccine development[2], [3], [4]. However, given the complex dynamics of disease spread, mathematical models that can help understand disease spread and control are becoming increasingly important.

Epidemiological models are used to predict the spread of disease and evaluate optimal control strategies. One of the challenges in building this model is parameter uncertainty and imperfect data. In this context, the Extended Kalman Filter (EKF) and Optimal Control theory can be used to improve the quality of predictions and identify the best intervention strategies. EKF allows estimation of parameters that change over time based on observational data, while Optimal Control helps

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determine efficient control policies to minimize the impact of the disease[5], [6], [7].

This article discusses the application of the Extended Kalman Filter and Optimal Control in the dengue spread model. By using this approach, it is hoped that the resulting model will be able to provide more accurate predictions and more effective control strategies in reducing dengue prevalence in the population.

To deal with this epidemic of dengue fever, common strategy for treating dengue fever is vector control. From the results of research conducted, preventative reduction of mosquito populations through vector control (reduction of larvae and insecticides) only delayed the outbreak [2], did not reduce the number of infections. This is in line with research on the use of insecticides[8]. Despite using vector control with community participation with active disease monitoring and insecticides, there was only limited success in preventing dengue fever and control on a national scale [9]. In addition, the use of insecticides can increase mosquito resistance levels[10]. As a result of the lack of specific treatment for dengue fever and limited treatment with vector control, vaccines are currently being developed as a form of treatment for dengue fever [3]. The vaccine is assumed to give temporary immunity, that is the resistant human can turn back to be susceptible. It is based on how dengvaxia vaccine that makes antibodies and protect against all four types but how long it last still on research.

The dengue fever model with vaccination as control model is a system of nonlinear differential equations in the form of Susceptible-Infected-Recovery (SIR) for human populations and Susceptible-Infected (SI) for mosquito populations. Initially, dengue with vaccination control was modeled as a constant control[2]. Subsequently, for a more realistic approach, the model was refined using optimal control theory. While some studies use the Aquatic-Susceptible-Infected (ASI) model for mosquito populations [11], his research adopts the SIR model for humans and the SI model for mosquitoes, focusing on preventing disease spread among humans rather than eradicating mosquito vectors.

$$
\begin{aligned}
\dot{s}_h &= \mu_h - (B\beta\frac{i_m N_m}{N_h} + \mu_h + u)s_h - \theta u r_h \\
\dot{i}_h &== B\beta\frac{i_m N_m}{N_h}s_h - (\eta_h + \mu_h)i_h \\
\dot{r}_h &= \eta_h i_h + u s_h - (\theta u + \mu_h)r_h \\
\dot{s}_m &= \mu_m - (B\beta i_h + \mu_m)s_m \\
\dot{i}_m &= B\beta i_h s_m - \mu_m i_m
\end{aligned} \tag{1}
$$

The adapted model in system (1) is a normalized form [12], the parameters and variables explained in Table 1 and Table 2.

In real life, due to various measurement limitations, not all variables in the dengue fever epidemic model can be measured. Therefore, a tool is needed to estimate unmeasured variables with known related variables. One method for estimating variables in nonlinear systems is the extended Kalman filter (EKF). The extended Kalman filter is a development of the Kalman Filter for nonlinear models by linearizing the system around the estimated Kalman filter point, which was originally proposed by Stanley Schmidt for the spacecraft problem.[13] In general, the Kalman filter estimates the system based on measurements with the aim of minimizing error covariance. In this study, an extended Kalman filter with discrete measurements of infected human will be used to estimate the state in the dengue fever model. Next, using optimal control will be sought in the form of vaccination combined with EKF designed to reduce the number of infections, that is infected human.

Method

Assuming that the dynamic system of the dengue fever model is ideal, and the measurements have disturbances, the steps taken are illustrated in the following flow diagram. Based on the flow diagram in Figure 1, there are two main steps in control with estimation, namely: estimation stage and control update. First, we will look at the state estimation or this dengue fever model. Then, from the estimation results, optimal control u^* will be sought using Optimal Control Theory.

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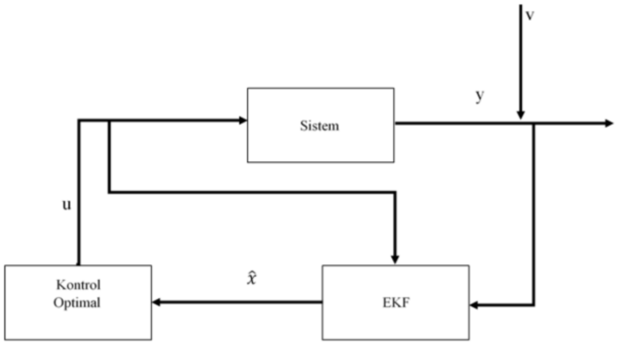

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Figure 1. Flowchart Optimal Control with EKF

Table 1. Variables of dengue model

Variable	Description
s_h	proportion of susceptible human
i_h	proportion of infected human
r_h	proportion of recovered human
s_m	proportion of susceptible mosquito
i_m	proportion of infected mosquito

Table 2. Parameters of dengue model

Parameter	Description	Unit
1	Average lifespan of humans	in days
$\frac{1}{\mu_m}$	Average lifespan of adult mosquitoes	in days
B	Average number of bites on humans by mosquitoes	bites per days
β_{hm}	Transmission probability from infected mosquitoes	per bites
β_{mh}	Transmission probability from infected humans	per bites
η_h	Human recovery rate	in days
θ	waning immunity process	proportion

Results And Discussion

State estimation with Extended Kalman Filter (EKF)

In general, the steps for estimating a state x with $\hat{x}$ using a continuous system EKF with discrete measurements are as follows[14].

Given a system of continuous and discrete measurements for each time t_k, x is state variable and input control u . $f(x, u, t)$ is state transition function, G represent the process noise transition matrix corrresponding with process noice $w(t)$. $h[x(t_k), k]$ is measurement function and v_k is measurement noise.

$$\dot{x} = f(x, u, t) + Gw(t)$$

$$y_k = h[x(t_k), k] + v_k$$

For the initial state value $x(0)$, with $(\bar{x}_0, P_0)$ s the initial state estimate and its covariance, the process noise $w(t) \in (0, Q)$, and the measurement noise $v_k \in (0, R)$ *are assumed to be* uncorrelated *white noise*.[15] The step of EKF is

1. Initialization

$$\hat{x}_0 = \bar{x}_0, P(0) = P_0$$

2. *Time Update*

Estimate

$$\dot{\hat{x}} = f(\hat{x}, u, t)$$

Error Covariance

$$\dot{P} = A(\hat{x}, u)P + PA^T((\hat{x}, t) + GQG^T$$

3. Measurement Update

Kalman gain

$$K_k = P(t_k)H^T(\hat{x}_k)[H(\hat{x}_k)P(t_k)H^T(\hat{x}_k) + R]^{-1}$$

Error Covariance

$$P(t_k) = [I - K_k H(\hat{x}_k)]P(t_k)$$

Estimate

$$\hat{x}_k = \hat{x}_k + K_k[y_k - h(\hat{x}_k, k)]$$

4. Jacobian

$$A(x, t) = \frac{\partial f(x, u, t)}{\partial x} \quad H(x) = \frac{\partial h(x, k)}{\partial x}$$

$A(x, t) = \frac{\partial f(x, u, t)}{\partial x}$ $H(x) = \frac{\partial h(x, k)}{\partial x}$Assuming
$H(x) = \frac{\partial h(x, k)}{\partial x}$Assuming

The Jacobian from system (2) is (2)

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$$A = \begin{bmatrix} -\dfrac{B\beta N_m i_m}{N_h} - \mu_h - u & 0 & \theta u & 0 & -\dfrac{B\beta N_m s_h}{N_h} \\ \dfrac{B\beta N_m i_m}{N_m} & -\mu_h - \eta_h & 0 & 0 & \dfrac{B\beta N_m s_h}{N_h} \\ u & \eta_h & -\mu_h - \theta u & 0 & 0 \\ 0 & -B\beta s_m & 0 & -B\beta i_h - \mu_m & 0 \\ 0 & B\beta s_m & 0 & B\beta i_h & -\mu_m \end{bmatrix}$$

Note that measured variable is infected human, so the measurement equation can be written as

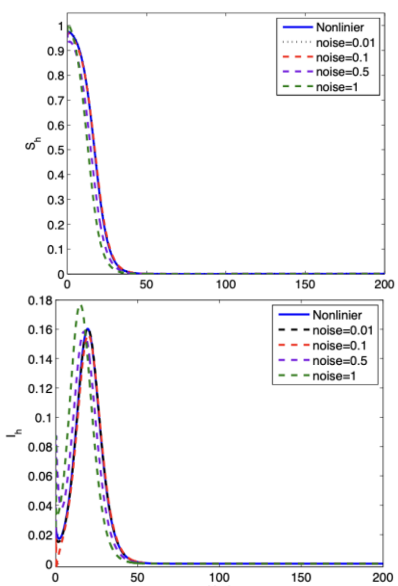
$$y_k = i_h + v_k$$

so the measurement jacobian is

$$H = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

We assumed the noise transition matrix G is a 5×5 identity matrix. The covariance matrix P represents the uncertainty in the state variables of a system. Each element of P describes how uncertain we are about the corresponding state variables, as well as how these uncertainties are correlated. Next, using the EKF method, the estimation results will be simulated with several standard deviations of disturbances.

Simulation



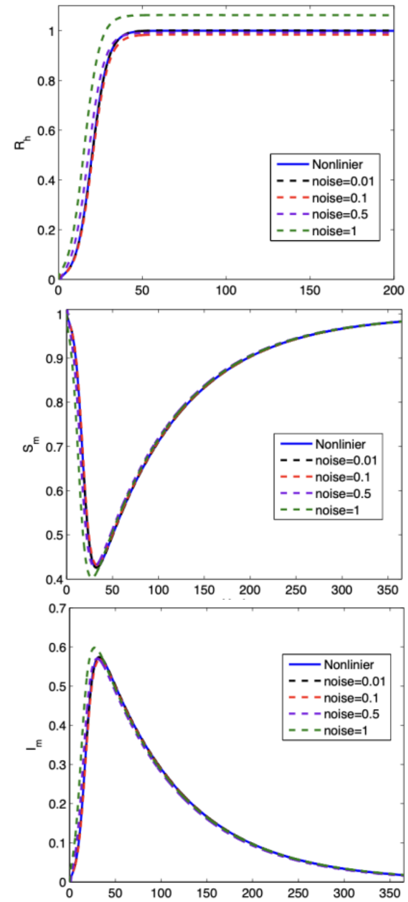


Figure 2. State Estimation with EKF

Suppose the human population is 10^5 people, so the data available can be adjusted into proportions. Parameters used for this simulation are $\frac{1}{\mu_m} = 90, B = 1$, $\frac{1}{\mu_h} = 71 \times 365$, $\beta_{hm} = 0{,}375$, $\beta_{mh} = 0{,}375$, $\eta_h = \frac{1}{3}$ [8], [11], [16]. The initial value $x_0 =$ [0.975; 0.025; 0; 1; 0] and the covariance matrix $P_0 = I_5$. In this simulation, four types of measurement noise v_k are used, that is 0.01, 0.1, 0.5, and 1. In the first simulation, the EKF was applied without vaccination control ($u = 0$) showed in Figure 2 for each compartment. The result show that the smaller the measurement noise the closer the estimation results align with the nonlinear system. Likewise with error covariance, the smaller the disturbance covariance, the faster the error covariance will go.to zero. From these findings, it can be concluded that state estimation using the EKF for the dengue fever model, without system

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interference, performs well in estimating each variable. This is true even when the measurements are affected by random noise within the selected measurement noise range.

Optimal Control with Extended Kalman Filter

The modified control design applies optimal control in conjunction with the Extended Kalman Filter (EKF). The optimal control u^* is determined through the forward-backward scheme[6] aiming to minimize the cost function $J[u]$. This approach utilizes the EKF to estimate the state variables, enabling the implementation of optimal control on the nonlinear model using the estimated states. Given the cost function

$$J[u] = \int_0^T [\gamma_D i_h^2 + \gamma_V u^2] dt \tag{3}$$

where T is the end time, $\gamma_D i_h^2$ and $\gamma_V u^2$ are quadratic functions that represent the burden of care for infected individuals and the burden of vaccination, respectively.

If $x = [s_h \quad i_h \quad r_h \quad s_m \quad i_m]^T$ and $\lambda = [\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5]^T$, Hamiltonian from system (2) and cost function (3) is

$$\begin{aligned}\mathcal{H}(x,\lambda,u) = \gamma_D i_h^2 + \gamma_V u^2 + \lambda_1\left(\mu_h - B\beta_{mh}\frac{i_m s_h N_m}{N_h} - \mu_h s_h - u s_h + \theta u r_h\right) + \\ \lambda_2\left(B\beta_{mh}\frac{i_m s_h N_m}{N_h} - (\eta_h + \mu_h) i_h\right) + \lambda_3(\eta_h i_h - \mu_h r_h + u s_h - \theta u r_h) + \\ \lambda_4(\mu_m - (B\beta_{hm} i_h + \mu_m) s_m) + \lambda_5(B\beta_{hm} i_h s_m - \mu_m i_m)\end{aligned}$$

State functions from Hamiltonian

$$\frac{\partial \mathcal{H}}{\partial \lambda_1} = \dot{s}_h, \frac{\partial \mathcal{H}}{\partial \lambda_2} = \dot{i}_h, \frac{\partial \mathcal{H}}{\partial \lambda_3} = \dot{r}_h, \frac{\partial \mathcal{H}}{\partial \lambda_4} = \dot{s}_m, \frac{\partial \mathcal{H}}{\partial \lambda_5} = \dot{i}_m.$$

Costate functions from hamiltonian

$$\begin{aligned}\dot{\lambda}_1 &= -\frac{\partial \mathcal{H}}{\partial s_h} = (\lambda_1 - \lambda_2) B\beta_{mh}\frac{i_m N_m}{N_h} + \lambda_1 \mu_h + (\lambda_1 - \lambda_3) u \\ \dot{\lambda}_2 &= -\frac{\partial \mathcal{H}}{\partial i_h} = -2\gamma_D i_h + \lambda_2(\eta_h + \mu_h) - \lambda_3 \eta_h + (\lambda_4 - \lambda_5) B\beta_{hm} s_m \\ \dot{\lambda}_3 &= -\frac{\partial \mathcal{H}}{\partial r_h} = -\lambda_1 \theta u + \lambda_3(\mu_h + \theta u) \\ \dot{\lambda}_4 &= -\frac{\partial \mathcal{H}}{\partial s_m} = (\lambda_4 - \lambda_5) B\beta_{hm} i_h + \lambda_4 \mu_m \\ \dot{\lambda}_5 &= -\frac{\partial \mathcal{H}}{\partial i_m} = (\lambda_1 - \lambda_2) B\beta_{mh} a \frac{s_h N_m}{N_h} + \lambda_5 \mu_m\end{aligned}$$

with transversality condition $\lambda_i = 0$ for $i = 1, \dots 5$.

Stationery condition

$$0 = \frac{\partial \mathcal{H}}{\partial u} = 2\gamma_V u - (\lambda_1 - \lambda_3)(s_h - \theta r_h) \leftrightarrow u = \frac{(\lambda_1 - \lambda_3)(s_h - \theta r_h)}{2\gamma_V}.$$

Control u represents the proportion of susceptible humans that one decides to vaccinate at time t . Parameter θ associated with control u represents the waning immunity process. It is assumed that for every time t, a θ proportion of vaccinated human came back to susceptible. Optimal control problem is constrained for $0 \le u \le 1$, so based on Pontryangin Minimum Principle, the optimal control u for $0 \le t \le T$ is determined to minimize cost function subject to the constraints and the system dynamics. The principle states that the optimal control must satisfy the following

$$\mathcal{H}(x^*, \lambda^*, u^*) \le \mathcal{H}(x^*, \lambda^*, u).$$

The result is characterized control as follows

$$u^* = \begin{cases} 0 & \dfrac{(\lambda_1 - \lambda_3)(s_h - \theta r_h)}{2\gamma_V} \le 0 \\ \dfrac{(\lambda_1 - \lambda_3)(s_h - \theta r_h)}{2\gamma_V} & 0 \le \dfrac{(\lambda_1 - \lambda_3)(s_h - \theta r_h)}{2\gamma_V} \le 1 \\ 1 & \dfrac{(\lambda_1 - \lambda_3)(s_h - \theta r_h)}{2\gamma_V} \ge 1 \end{cases}$$

Furthermore, the control u can be written as

$$u^* = \min\left(1, \max\left(0, \frac{(\lambda_1 - \lambda_3)(s_h - \theta r_h)}{2\gamma_V}\right)\right).$$

Optimal control with EKF computed as follows

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1. Choose the initial value $u(0)$
2. Use the Extended Kalman Filter (EKF) to estimate the state function $\hat{x} = [\hat{s}_h \quad \hat{\imath}_h \quad \hat{r}_h \quad \hat{s}_m \quad \hat{\imath}_m]^T$
3. Compute the costate function backward with estimated states $\hat{x}$
4. Update control u by subtitute states and costate values
5. Convergence checking, if error is small, control is optimal, if not, back to step 2.

Simulation

Forward-backward simulation using the parameter on EKF simulation with end time $T = 365$. The simulations were carried out using a chosen θ =0.05 that represents proportion of resistant who come back to susceptible at time t, performance index $\gamma_D i_h^2 = 0.5$, $\gamma_V u^2 = 0.5$ and $v_k = 0.01$. The result shown n Figure 4. Using EKF and Optimal control, it can be seen from Figure 3 and Figure 4 that the optimal control obtained is a decreasing function and with this control the size of the infected population can be reduced from 0.1604 to 0.01997. This optimal control cost is 0.0565501731501846. Combining an Extended Kalman Filter with optimal control in a dengue model offers a robust approach for dynamically managing the disease. By providing state estimates and compensating for noisy and uncertain data, it enables the design of more adaptive. Next, a comparison of the reduction in the proportion of infected humans and optimal control for different standard deviations of v_k disorders will be studied. The optimal cost results for each standard deviation of disturbance are shown in Figure 5.

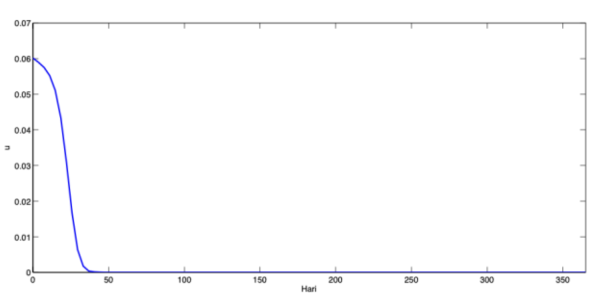

Figure 3. Optimal Control

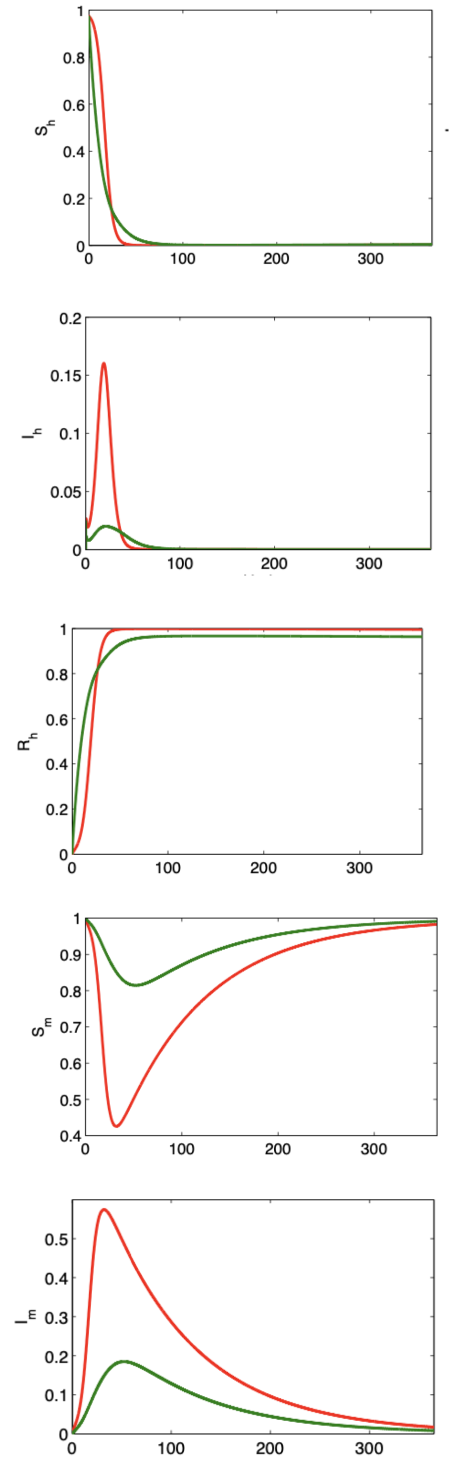

Figure 4. Comparison of Optimal control with EKF and without control with EKF

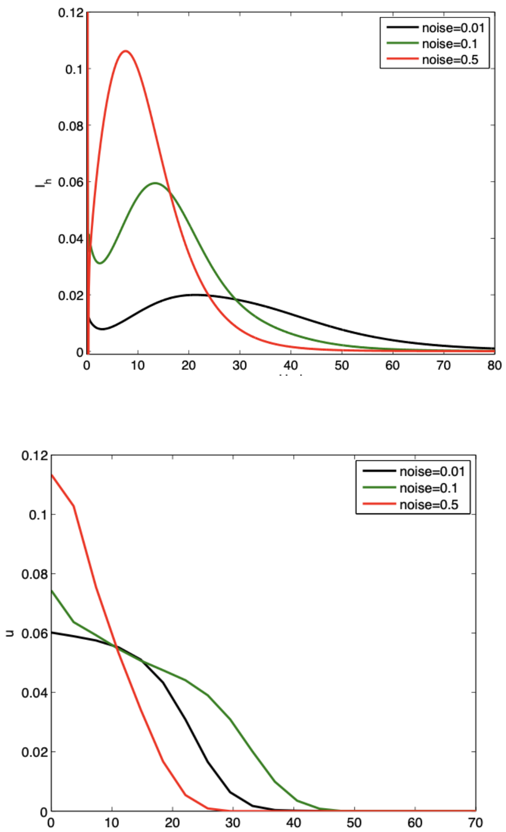

Figure 5. Comparison of infected human and optimal control compartments for different noise standard deviations

From Figure 5, the smaller the standard deviation of the disturbance, the smaller the optimal control required at the start. With a larger standard deviation of measurement noise, the human compartment infected after being given optimal control is greater. This can be caused by jumping at the beginning of the iteration with EKF due to the large standard deviation of disturbances, so that the estimation results for the infected human compartment are higher, as a result the resulting optimal control is also greater at the beginning.

Conclusions

This research shows that the application of the Extended Kalman Filter (EKF) and Optimal Control in the dengue fever spread model can provide significant benefits in addressing lighting parameters and determining effective control strategies. Through EKF, states that are difficult to measure directly can be estimated dynamically based on continuously updated observation data, thereby increasing the accuracy of predicting the spread of disease. On the other hand, Optimal Control allows designing optimal interventions, namely vaccination, which can minimize the number of infection cases in a cost-efficient manner. The simulations carried out prove that the combination of these two methods is able to produce better solutions than conventional approaches, especially in conditions where the dynamics of disease spread are strongly influenced by external factors such as the environment and intervention policies. It is hoped that the results of this research can help policy makers and public health practitioners in designing more adaptive and data-based dengue control strategies, so that they can reduce the rate of spread of the disease and reduce the overall economic and public health impact. Overall, this research reinforces the importance of using modeling and optimization-based methods to face the challenges of infectious diseases such as dengue which continue to grow and require flexible and timely approaches.

Conflicts of interest

There are no conflicts to declare.

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References

[1] "World Health Organization (WHO." [Online]. Available: https://www.who.int/news-room/fact-

[2] M. Derouich, A. Boutayeb, and E. H. Twizell, *A Model of Dengue Fever*. BioMedical Engineering Online, 2003.

[3] S. Murrel, S. C. Wu, and M. Butler, "Review of Dengue Virus and The Development of a Vaccine," *Biotechnol Adv*, vol. 29, no. 2, p. 239, 2011.

[4] D. C. P. P. P. Ch, "Tropical Disease Lacking Adequate Control Measures: Dengue, Leishmaniasis, and African Tripanosomiasis." pp. 452–466.

[5] S. Lenhart and J. T. Workman, *Optimal Control Applied to Biological Models*. Chapman & Hall/CRC Taylor and Francis Group, 2007.

[6] F. L. Lewis, D. L. Vrabie, and V. L. Syrmos, *Optimal control*, Third. New York: John Wiley & Sons, 2012.

[7] K. D. Hammet, *Control of Nonlinear Systems via State Feedback State- Dependent Riccati Equation Techiniques*. Disertasi. Air Univerisity, 1997.

[8] E. A. dan R. Newton and P., "A Model of The Transmission of Dengue Fever with An Evaluation of The Impact of Ultra-low volume(ULV) Insecticide Application on Dengue Epidemics," *Am J Trop Med Hyg*, vol. 47;709-720, 1992.

[9] P. Cattand and dkk, "Disease Control Priorities in Developing Countries." 2006.

[10] M. J. dan R. Keeling and P., *Modeling Infectious Disease in Human and Animals*. Princeton. Princeton University Press, 2008.

[11] H. S. Rodrigues, M. T. T. Monteiro, and D. F. M. Torres, "Vaccination Models and Optimal Control Strategies to Dengue," *Mathematical Biosciences*, vol. 247, pp. 1–12, 2013.

[12] H. H. Aulia, R. Saragih, and D. Handayani, "Control Design for Dengue Fever Model with Disturbance," *Communication in Biomathematical Sciences*, vol. 5, no. 2, Art. no. 2, 2022, doi: 10.5614/cbms.2022.5.2.3.

[13] S. F. Schmidt, "The Kalman filter - Its recognition and development for aerospace applications," *Journal of Guidance and Control*, vol. 4, no. 1, pp. 4–7, Jan. 1981, doi: 10.2514/3.19713.

[14] K. Zhou and J. C. Doyle, *Essentials of Robust Control*. Prentice Hall, 1998.

[15] F. L. Lewis, *Optimal and Robust Estimation: With an Introduction to Stochastic Control Theory, Second Edition*.

[16] Z. A. Leleury, Y. A. Lesnussa, J. B. Bension, and Y. S. Kakisina, "Anali- sis Stabilitas Model SIR (Susceptible, Infected, Recovery) pada Penyebaran Penyakit Demam Berdarah Dengue di Maluku," *Jurnal Matematika*, vol. 7, no. 2, p. 158, 2017.