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RESEARCH ARTICLE

Seismic Refraction Modeling Using Time-Terms Inversion Based on Linear Programming: A Study on Synthetic Data

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Abstract Seismic refraction is a geophysical method widely used to model the distribution of subsurface velocity. The main challenge of this method is the instability of the inversion that often arises due to noise and inaccuracies in determining the first arrival time (*first arrival picking*). In general, the inversion is carried out using the *Least-Squares* method, which tends to be unstable under non-ideal data conditions. To overcome this, this study develops a *Time-Terms Inversion* (TTI) approach utilizing *Linear Programming*. The linear formulation of TTI allows the inversion process to be expressed as an optimization problem with a clearly defined objective function, so that the resulting solution is more stable and resistant to data errors. Numerical tests on synthetic data show that this approach can reduce the influence of picking errors while maintaining the accuracy of the velocity model. These results show that the combination of TTI and *Linear Programming* is effective for seismic refraction modeling under synthetic data conditions affected by noise and picking inaccuracies.

1. Introduction

The seismic refraction method is one of the geophysical engineering methods widely used to obtain information on the velocity of subsurface layers. This technique is generally applied in geotechnical studies, resource exploration, and environmental research because it can provide an overview of the distribution of seismic wave velocity relatively quickly and efficiently (Dobrin & Savit, 1988; Sheriff & Geldart, 1995; Abdelgowad et al., 2025; Odoh et al., 2025). This velocity information is very important for understanding lithological characteristics, the depth of the bedrock (*bedrock*), as well as shallow subsurface conditions that often become the main focus in civil engineering studies and resource exploration.

In practice, there are several methods used to model seismic refraction data. Methods based on simple travel-time curves are still often used for rapid interpretation, but their results are limited to homogeneous layer models (Hagedoorn, 1959). For more detailed modeling, the tomographic approach offers high resolution, but requires dense data and a complex computational process (Zelt & Smith, 1992; Watts et al., 2023). Between these two approaches, Time-Terms Inversion (TTI) emerges as a relatively simple yet sufficiently effective method. TTI works by formulating travel time as a combination of the layer velocity parameters and the time-term at the refraction point (Willmore, 1960; Iwasaki, 2002; Al-Heety, 2018; Guedes et al., 2022). This method has proven efficient in providing initial estimates of subsurface velocity (Goodwin et al., 2016; Akingboye & Abudulraheem, 2025).

Although TTI is fairly easy to implement, the inversion process is usually carried out using the Least Squares (LS) approach. The LS method works by minimizing the squared difference between the measured data and the model data. However, this approach is very sensitive to noise and errors in determining the first arrival time (*first arrival picking*) (Poormirzaee, 2022; Wen et al., 2025). Under field-data conditions that are often contaminated by noise, the LS solution can become unstable, difficult to converge, or produce a biased velocity model. Various modern studies also show that more robust inversion methods such as linear programming can provide more stable results (Penta de Peppo et al., 2024; Ai et al., 2025; Huang et al., 2021).

To overcome these limitations, this study proposes the application of Linear Programming (LP) to the TTI formulation. Because TTI can be derived in a linear form, the travel-time equation can be expressed as an optimization problem with a clear objective function. With the LP framework, the inversion solution becomes more resistant to noise and picking errors, while still maintaining the simplicity of TTI (Bird, 1982; Law, 2017; Rücker et al., 2017; Zeyen & Schlindwein, 2024).

Based on the description above, this study aims to test the effectiveness of the LP-based TTI approach on synthetic data. The research focus is directed at evaluating the stability and accuracy of the method under data conditions contaminated by noise and picking errors. Thus, this study not only tests the reliability of LP in TTI modeling, but also emphasizes its contribution as an alternative inversion method that is more robust than the conventional Least Squares-based approach.

2. Data and Methods

2.1 Generation of Synthetic Data

To test the performance of the *Time-Terms Inversion* (TTI) approach based on *Linear Programming* (LP), this study uses synthetic data constructed from a simple velocity model in the form of horizontal layers. The synthetic model was chosen to facilitate the evaluation of the stability of the inversion algorithm under controlled conditions, while also providing an initial overview of the effectiveness of the method before it is applied to field data.

The synthetic model consists of two horizontal layers with different velocities. The first layer has a velocity of 1000 m/s, while the second layer has a velocity of 1700 m/s. The two layers are separated by a constant depth of 10 meters. This horizontal configuration was chosen because it is simple yet representative enough to test the stability of the LP algorithm when facing noise variations.

Data acquisition was carried out using three shot points placed evenly on the surface. Each shot point records seismic waves captured by a series of receivers at varying positions on the surface; the synthetic data parameter information is given in Table 1.

To simulate more realistic field conditions, the synthetic data were tested not only under ideal conditions, but also with the addition of random Gaussian noise. This noise was added to the travel-time domain (*travel time*) with a standard deviation of 5% and 10% of the actual travel-time value. Not all data were given noise; rather, 10-30% of the total data were randomly selected to be given noise. The wave arrival-time values are given in Table 5.

This approach aims to evaluate the robustness of the *Linear Programming* (LP) algorithm in facing data uncertainty, particularly due to first-arrival picking errors and the presence of signal disturbances during field acquisition. By testing two disturbance levels (5% and 10%), this study can observe the extent to which LP is able to produce a stable solution compared with the conventional inversion method (*least-squares inversion with the normal-equation approach*).

Table 1 Synthetic Data Parameters

Parameter	Value	Description
Number of layers	2	Horizontal model
Velocity of layer 1	1000 m/s	Upper layer
Velocity of layer 2	1700 m/s	Lower layer
Depth of boundary layer	10 m	Constant, horizontal
Number of shot points	3	Placed evenly on the surface
Number of geophones	48	Placed evenly on the surface
Spacing between geophones	5	-
Noise variation (standard deviation for Gaussian noise)	5% and 10%	Added to the arrival time (*travel time*)

Computational Implementation

The inversion process of *Time-Terms Inversion* (TTI) based on *Linear Programming* in this study was implemented using the Python programming language with the support of the NumPy library for numerical computation and SciPy for solving the optimization problem of *Linear Programming* (Virtanen et al., 2020). In general, the implementation stages can be described as follows:

The following pseudo-code illustrates the computational flow of the inversion of *Time-Terms Inversion* (TTI) with the *Linear Programming* approach, given in Table 2.

Table 2 Pseudo-code for modeling with Linear Programming.

INPUT:

```
A  ∈ ℝ^{M×N}       # TTI system matrix
d  ∈ ℝ^{M}         # arrival-time data vector
Nstn                # number of stations
bounds              # depth and slowness bounds
```

VARIABLE DEFINITION:

```
x = [ τ_1, …, τ_{Nstn}, p ]
e = [ e_1, …, e_M ]
```

COMBINE VARIABLES:

```
z = [ x ; e ]
```

RESIDUAL CONSTRAINTS (−e ≤ A x − d ≤ e) → form A_ub z ≤ b_ub:

```
  A_ub = [  A   -I]
         [ -A   -I]
  b_ub = [  d ]
         [ -d ]
```

LP SOLVING:

```
  result = linprog( c, A_ub=A_ub, b_ub=b_ub, bounds=bounds,
                    method="highs" )
```

RESULT EXTRACTION:

```
  x_est = result.z[0:N]
  τ_est = x_est[0:Nstn]
  p_est = x_est[Nstn]
  v_est = 1 / p_est
```

OUTPUT:

`τ_est, p_est, v_est,` as well as the misfit value |A x_est − d|

z computed from `τ_est`

3. Results and Discussion

The Results section should present the main findings of the study in a clear, logical, and objective manner. Begin by describing the overall patterns or trends observed in the data before focusing on more specific details. Each result should directly relate to the research objectives or hypotheses outlined earlier. Figures, tables, and maps should be used effectively to illustrate key outcomes, and every visual element must include a comprehensive and self-contained caption that allows readers to interpret it independently.

Next, highlight the quantitative results, relationships, or model outputs that represent the core contributions of the study. Avoid repeating numerical values from tables in the text; instead, emphasize interpretations and comparisons that reveal important features or contrasts. When appropriate, describe statistical measures, model performance indicators, or sensitivity outcomes to demonstrate the reliability and significance of the results. Maintain an objective tone, avoiding speculation or extended interpretation, which should be reserved for the Discussion section.

Finally, organize the presentation of results in a logical sequence, such as from general observations to detailed analyses, or from input data to derived parameters. Subsections can be used to group related findings for clarity. Where relevant, include comparisons among datasets, model scenarios, or experimental conditions to strengthen the narrative flow. The ultimate goal of this section is to provide a complete and coherent picture of the study's findings without yet interpreting their broader implications.

Model Test without Noise

The first test was carried out on synthetic data without the addition of noise. Both the Least-Squares (LS) method, solved through the normal-equation approach, and Linear Programming (LP) were able to model the subsurface very well. Both methods successfully estimated the depth of the second layer in accordance with the synthetic model and provided refractor velocity values close to the actual conditions (Figure 1).

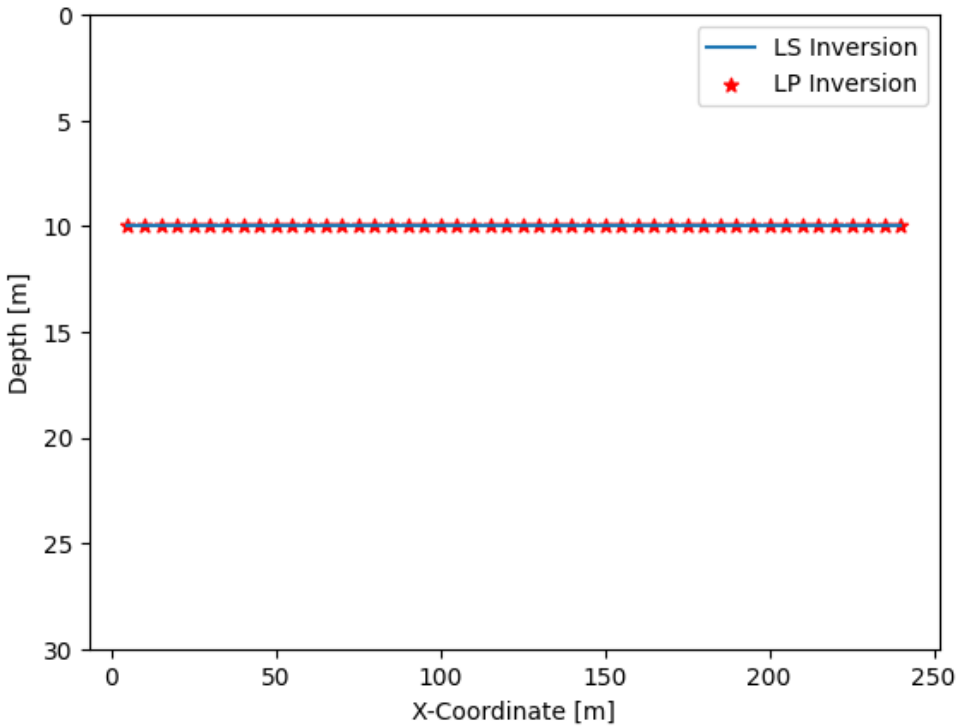

*Figure 1 Reconstruction results of the subsurface model with noise-free data, using LP (red * markers) compared with LS (blue line).*

Under these ideal conditions, there is no significant difference between the LS and LP inversion results. This shows that under clean-data conditions, both methods can equally be used for seismic refraction modeling. LS provides a fast solution with the least-squares approach, while LP is able to provide results consistent with the minimization formulation of the norm-l_1. Thus, it can be concluded that on data without noise, both LS and LP are equally accurate and stable.

Model Test with Noise

In the testing stage with data to which Gaussian noise disturbances of 5% and 10% of the travel time had been added, a fairly clear difference between the LS and LP methods can be seen. The presence of noise in the data represents more realistic field conditions, where errors in *picking first arrival* or disturbances from acquisition quality often cannot be avoided.

The inversion results show that the LS method becomes less accurate in modeling the subsurface. This occurs because the squared-norm nature of LS gives large residuals a more dominant weight, so that one or several errors in the data can pull the solution away from the true value. As a result, the estimates of the second-layer depth and the refractor velocity obtained by LS deviate quite far from the synthetic model, especially for fairly large data disturbances (10% noise) (Figure 2).

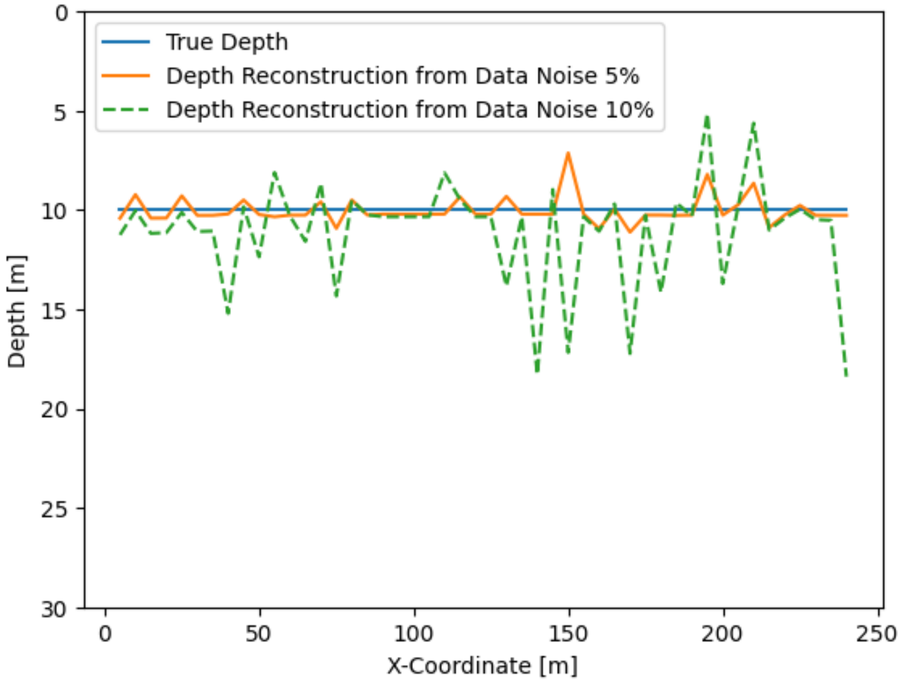

Figure 2 Reconstruction results of the subsurface model with LS modeling.

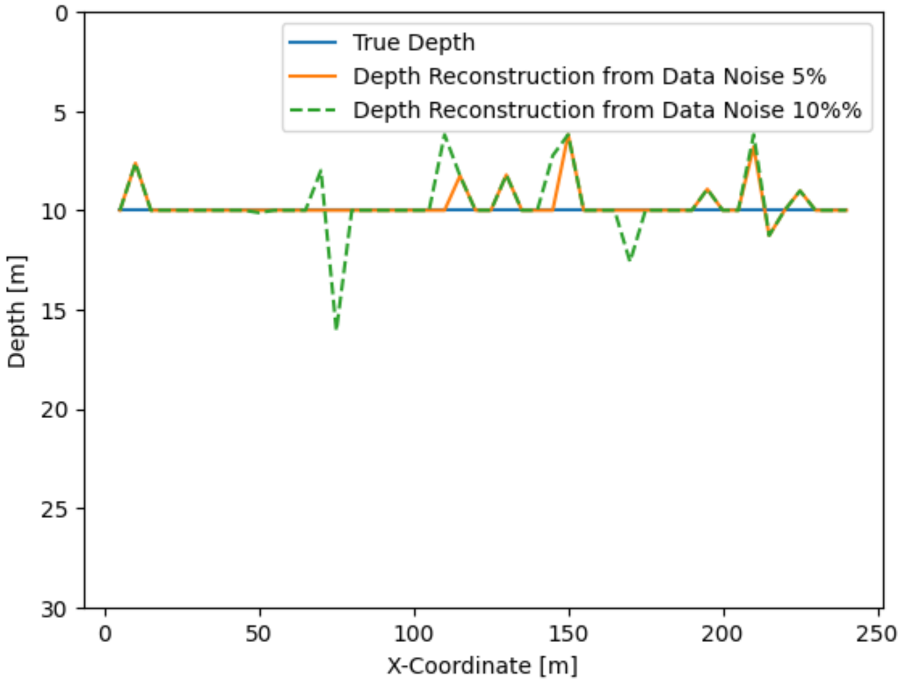

Figure 3 Reconstruction results of the subsurface model with LP modeling.

In contrast, the LP method, which uses the ℓ1-norm minimization approach, provides more stable results. With the absolute norm, the influence of large residuals does not dominate the solution, so that the inversion parameters are more robust to disturbances. The layer-depth estimate remains consistent with the synthetic model, and the refractor velocity value obtained is closer to the actual conditions (Table 3) than the LS result; the subsurface model from the LP method is given in Figure 3.

Table 3 Comparison of the second-layer velocity values using the LS and LP methods.

Data	LS (m/s)	LP (m/s)
Noise 5%	1702.0	1700.0
Noise 10%	1710.0	1700.0

Table 4 Comparison of reconstruction results for noise-free and noisy data using LS and LP.

Scenario	**Method**	**ΔDepth (abs)**	**ΔVelocity(V_2) (abs)**	**Note**
Without noise	LS	no difference	no difference	Both accurate (baseline)
Without noise	LP	no difference	no difference	Equivalent to LS
Noise 5–10%	LS	increases	increases	Affected by *outlier*
Noise 5–10%	LP	smaller	smaller	Robust and stable

These findings show that although both methods successfully reconstruct the subsurface model on clean data, on noisy data only the LP approach is able to maintain accuracy; the advantages of the LP method are presented in Table 4. Thus, the use of Linear Programming-based TTI can be considered more suitable for dealing with field data that are generally non-ideal and laden with uncertainty.

4. Conclusions

The research results show that Linear Programming (LP) is superior in seismic refraction modeling, especially when the data contain noise. On clean data, both LP and Least-Squares (LS) are equally able to reconstruct the depth and refractor velocity in accordance with the synthetic model. However, when noise is added, LS becomes less stable and accurate because it is sensitive to large residuals, whereas LP remains robust and consistent thanks to the $\ell 1$-norm approach. Thus, LP can be considered a more suitable and reliable method for dealing with field-data conditions that are generally full of uncertainty, while LS serves only as a comparison under ideal conditions.

Nevertheless, this study is still limited to synthetic tests with a simple model. Therefore, further testing is needed on data with more complex geometry, including undulating layer conditions and actual field data, so that the validity and reliability of the LP method can be confirmed in more realistic modeling scenarios.

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